# MATHEMATICS

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## AIM POINTS MATHEMATICS

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TARGET IIT JEE AIEEE
& COMPATETIVE EXAM FOR XII (PQRS)

# CONTINUITY, DIFFERENTIABILITY AND DIFFERENTIATION

## CONTENTS

# CONTINUITY AND DIFFERENTIABILITY

$\lim_{h \to 0} f(a-h) = \lim_{h \to 0} f(a+h) = f(a)$

1. Is $\phi(x)$ continuous at x = 1, when

   $\phi(x) = x+3, \quad 0 \leq x \leq 1$

   $= 4-x, \quad x>1$ ?

2. Prove that the following function is discontinuous at x = ½ ?

   $f(x) = \frac{1}{2} - x, \quad 0 < x < \frac{1}{2}$

   $= \frac{1}{2}, \quad x = \frac{1}{2}$

   $= 3/2 - x, \quad \frac{1}{2} < x < 1$ ?

3. Examin the contunuity of the following fraction x = 0.

   $f(x) = \dfrac{\text{Cos ax} - \text{Cos bx}}{X^2}, x \neq 0$

   $= \dfrac{b^2 - a^2}{q}, x = 0$

4. Is f(x) continuous at x = 0, if

   $f(x) = x \sin \frac{1}{x}, \quad x \neq 0$

   $= 0, \quad x = 0$

5. Examine the contunuity of the following function f(x) in the closed interval $[0, \pi]$.

   $f(x) = \text{Sin } x + \text{Cos } x, 0 \leq x\ \pi/3$

   $= 2 \text{ Sin } 2x, \frac{\pi}{3} < x < 2\pi/3$

   $= \tan x, 2\pi/3 \leq x \leq \pi$

6. If $f(x) = x+5, x \leq 1$

   $= x - 5, x > 1,$ then Examin the countinuity of f(x).

7. Find the value of K so that the function

   $f(x) = Kx + 1, x \leq 5$

   $= 3x - 5, \quad x > 5$ is countinous at x = 5.

8. Find the value of K, So that the function is contunous at $x = \pi/2$

$$f(x) = \frac{K \cos x}{\pi - 2x}, \qquad x \le \pi/2$$

9. Examine the contunuity of the following function f(x), at x = 0 ?

$$f(x) = \frac{|x|}{x}, \qquad x \neq 0$$

$$= 0, \qquad x = 0$$

10. The function $f(x) = \frac{\log(1+ax) - \log(1-bx)}{x}, x = 0$

is not defined at x = 0, Find the value of f(0). So that f(x) is continuous at x = 0.

11. If $f(x) = \frac{\tan(\pi/4 - x)}{\cot 2x}$, $x \neq \pi/4$ then find $f(\pi/4)$ so that f(x) is continous at $x = \pi/4$.

## DIFFERENTIATION

Thins to remember

1. Deffernital co-efficient of standard function.

(i) $\frac{dx^n}{dx} = nx^{x-1}$

(ii) $\frac{d(c)}{dx} = 0$, where C = Constant

(iii) $\frac{d \sin x}{dx} = \cos x$

(iv) $\frac{d \cot x}{dx} = -\text{Cosec}^2 x$

(v) $\frac{d \cos x}{dx} = -\text{Sin}x$

(vi) $\frac{d \text{ cosec } x}{dx} = -\text{Cosec}x.\text{Cot}x$

(vii) $\frac{d \tan x}{dx} = \text{Sec}^2 x$

(viii) $\frac{d \text{ Sec } x}{dx} = \text{Sec}x.\tan x$

(ix) $\frac{de^x}{dx} = e^x$

(x) $\frac{d \log x}{dx} = \frac{1}{x}$

(xi) $\frac{d\sqrt{x}}{dx} = \frac{1}{2\sqrt{x}}$

2. (i) $\frac{d \text{ Sin}^{-1} x}{dx} = \frac{1}{\sqrt{1-x^2}}$

(ii) $\frac{d \text{ Cot}^{-1} x}{dx} = \frac{1}{\sqrt{1-x^2}}$

(iii) $\frac{d \tan^{-1} x}{dx} = \frac{1}{1+x^2}$

(iv) $\frac{d \text{ Cot}^{-1} x}{dx} = \frac{-1}{1+x^2}$

(v) $\frac{d\sec^{-1}x}{dx} = \frac{1}{x\sqrt{x^2-1}}$ (vi) $\frac{d\,\text{cosec}^{-1}x}{dx} = \frac{-1}{x\sqrt{x^2-1}}$

(vii) $\frac{da^x}{dx} = a^x.\log e^a$ (viii) $\frac{dx}{dx} = 1$

3. (i) If $y = u.v$ then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

(ii) If $y = u+v$ then $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

(iii) If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

4. Geometrical uses of $\frac{dy}{dx}$.

(i) The change Rate of $x = \frac{dx}{dt}$

(ii) $\frac{dy}{dx} = \frac{\text{Change rate of y}}{\text{Change Rate of x}}$

(iii) Velocity $v = \frac{ds}{dt}$

(iv) Acceleretion $f = \frac{dv}{dt} = \frac{d^2s}{dt^2}$ where s = displacement

(v) $\delta y = \frac{dy}{dx} . \delta x$ where $\delta y$ = y about change $\delta x$ = x, in about change.

5. (i) Any curve $y = f(x)$, for $\frac{dy}{dx} = \tan\psi$, where $\psi$ = Point of curve (x,y) at tangent the slope of x-axis.

(ii) The Equation of tangent at point $(\alpha,\beta)$ the curve $y = f(x)$

$$y - \beta = \left(\frac{dy}{dx}\right)_{\alpha,\beta} . (x - \alpha)$$

(iii) Equation of Normal at Point $(\alpha,\beta)$ the curve $y = f(x)$

$$(y-\beta)\left(\frac{dy}{dx}\right)_{\alpha,\beta} + (x - \alpha) = 0$$

(iv) If the curve $y = f_1(x)$, $y = f_2(x)$ and their between angle be $\phi$ then

$$\tan\phi = \frac{\pm\left(\frac{df_2}{dx}\right)_{\alpha,\beta} - \left(\frac{df_1}{dx}\right)_{\alpha,\beta}}{1+\left(\frac{df_1}{dx}\right)\cdot\left(\frac{(df_1)}{dx}\right)_{\alpha,\beta}}$$

Where $(\alpha,\beta)$ are cut - point.

6. Introduct of Maximum and Minimum Value.

(i) For Max value $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$

(ii) For min value $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$

7. (a) Theorem of Rolle's :-

(i) f(x) is continuous at every point of the closed interval $a \le x \le b$.

(ii) f'(x) exists at every point of the openinterval $a < x < b$ and

(iii) f(a) = f(b)

Then there is at least one value of c of at which f'(c) = 0, where $a<c<b$.

(b) Lagrange's mean value theorem :-

If the function f(x) be defined in such a way that

(i) f(x) is continuous at every point of the closed interval $a \le x \le b$.

(ii) f'(x) exists at every point of the open interval $a < x < b$.

then there is a value c of x for which,

$$\frac{f(b) - f(a)}{b-a} = f'(c) \text{ where } a<c<b.$$

Extra Formula

(1) $(a+x)^n = 1 + nx + \frac{n(n-1)}{3!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots\ldots\ldots\ldots\infty$

(2) $\log(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \ldots\ldots\ldots\ldots \infty$

(3) $e^x = 1 + x + \frac{x^2}{<2} + \frac{x^3}{<3} + \ldots\ldots\ldots\ldots$

(4) (i) Area of circle = $\pi r^2$, (ii) Perimeter of circle = $2\pi r$

(5) (i) Volume of Sphere = $4/3\pi r^3$ and their surface = $4\pi r^2$.

(6) Volume of cylinder = $\pi r^2 h$ and curve surface = $2\pi rh$.

(7) Volume of cone = $1/3\ \pi r^2 h$ and curve surface = $\pi rl$

# EXERCISE - 2

Differentiability :-

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

1. Examine the continuity and differentiability of the function f defined by

$$f(x) = \begin{cases} 2x + 3, & 3 > x < -2 \\ x + 1, & -2 \le x < 0 \\ x + 2, & 0 < x \le 1 \end{cases}$$

2. If $f(x) = \frac{x-2}{x^2-3x+2}$, When $x \neq 2$

$= 1$, when $x = 2$

then find the value of $f'(2)$.

3. Find the value of a and b so that

$f(x) = x^2 + 3x + a, \ x \le 1$

$= bx + 2, \quad x > 1$ is differentiable at $x = 1$.

# EXERCISE - 3

1. Find $\frac{dy}{dx}$

(i) $y = \text{Cos}\sqrt{\text{Sin}\sqrt{x}}$ (ii) $\text{Sin}\sqrt{x^2+ax+1}$

(iii) $y = \log(x+\sqrt{x^2+a^2})$ (iv) $y = \text{Sin}\{\text{Cos}(\tan\sqrt{x})\}$

(v) $y = e^{\sqrt{x}}$ (vi) $y = e^{x2}$

(vii) $y = \log \text{Sin}\, x$ (viii) $y = \log(\text{Sec}\, x + \tan x)$

(ix) $y = \log \tan(\pi/4 + x/2)$

2. If $x^m y^n = (x+y)^{m+n}$ then find $\frac{dy}{dx} = ?$

3. If $y = x \operatorname{Sin}(a+y)$ then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.

4. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

5. If $y = \sqrt{x+\sqrt{x+\sqrt{x+\ldots\ldots\text{to } \infty}}}$ then find $\frac{dy}{dx}$ ?

6. Prove that $\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\operatorname{Sin}^{-1}\frac{x}{a}\right] = \sqrt{a^2-x^2}$

7. Find $\frac{dy}{dx} = ?$ If $y = \tan^{-1}(\sqrt{1+x^2} + x)$ ?

8. If $9x^2 + 2hxy + by^2 + 2fy + c = o$, Find $\frac{dy}{dx} = ?$

9. If $(x+y)^{m+n} = x^m y^n$. Show that $\frac{dy}{dx} = \frac{y}{x}$

10. If $y = x + \frac{1}{x}$ then prove that $x\frac{dy}{dx} + y = 2x$.

11. If $y = \log_7 . \log_7 x$ Find $\frac{dy}{dx}$ ?

12. If $x^y = y^x$ then find $\frac{dy}{dx}$ ?

13. $y = x^{x^x}$ then find $\frac{dy}{dx}$ ?

14. $y = e^{x+e^{x+\text{---- to } \infty}}$ then prove that $\frac{dy}{dx} = \frac{y}{1-y}$

15. If $y = (\sin x)^x + (\cos x)^{\tan x}$ then find $\frac{dy}{dx} = ?$

16. If $y = (\log x)^x + x^{\log x}$ then find $\frac{dy}{dx} = ?$

17. If $(\cos x)^y = (\cos y)^x$ then find $\frac{dy}{dx}$ ?

18. If $Y = (\sin x)^{\cos x} + (\cos x)^{\sin x}$ then find $\frac{dy}{dx}$ ?

# Exercise :- 4

1. If $x = \log t + \sin t$, $y + e^t + \text{Cost}$ then $\frac{dy}{dx} = ?$

2. If $x = a \sin 2t\,(1 + \cos 2t)$ and $y = b \cos 2t\,(1 - \cos 2t)$ then show that $\left(\frac{dy}{dx}\right)_{t=\pi/4} = b/a$

3. If $y = \cos^2\theta$, $x = b \sin^2\theta$, then prove that $dy/dx + a/b = 0$

4. Find $dy/dx$, at $t = \pi/3$ when $x = a\cdot(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$

5. If $x = 3at/1+t^3$, $y = 3at^2/1+t^3$, find $dy/dx$ when $t = ½$

6. If $xy = e^{x-y}$ + then find $dy/dx = ?$

# Exercise - 5

1. $y = Ae^{mx} + Be^{nx}$, show that $d^2y/dx^2 - (m+n)\,dy/dx + mny = 0$

2. If $y = \sin^{-1}x$, then prove that $(1-x^2)\,d^2y/dx^2 - x\,dy/dx = 0$

3. Find the second order derovatives.

   (a) $\log(\log x)$ (b) $e^x \sin 5x$ (c) $x^3 \log x$

4. If $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$, prove that $d^2y/dx^2 = \sec^3\theta/a\theta$

5. If $y = 3\cos(\log x) + 4 \sin(\log x)$, prove that $x^2y_2 + xy_1 + y = 0$.

6. If $y = 3e^{2x} + 2e^{3x}$, prove that $d^2y/dx^2 - 5\,dy/dx + 6y = 0$.

7. If $y = a \cos(\log x) + b \sin(\log x)$, then prove that $x^2y_2 + xy_1 + y = 0$.

8. If $y = \log (x/a+bx)^x$ then prove that $x^3\, d^2y/dx^2 = (x\, dy/dx - y)^2$.

9 (a) Find the equation of Tangent by geome trical method ?

(b) Find the equation of normal by Geome trical method ?

10. (a) At which point on the curve $\sqrt{x} + \sqrt{4} = 4$, the tangent is equally inclined with the axes ?

(b) Prove that the tangent to the curve $y^2 = 2x$ at the points where $x = ½$ are at right angles.

(c) What is the condition for the curves $ax^2+by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ to cut

or thogonally ?

11. Find the approximate volume of the metal of a hollow spherical shell whose thickness is 0.01 mec. and internal radius is 5 inch.

12. Find the approximate value of $\cos 31^0$ and $\cos 29^0$. If $1^0 = 0.017$ radian.

13. Find the approximate $(80)^{1/4}$

14. Prove that $f(x) = x^3 - 3x^2 + 4x + 3$ is an incresing function at all points of its domain.

15. Prove that $\sin\theta\,(1+\cos\theta)$ has maximum value of $\theta = \pi/3$

16. Prove that the semi-vertical angle of a cone with given slant height, whose volume is maximum is $\tan^{-1}\sqrt{2}$

17. Prove the the rectangle of greatest area incribed in a circle is a square.

18. If $\theta+\varphi=\alpha$ which is constant, prove that $\sin\theta \, . \sin\varphi$ has maximum value when $\theta = \varphi$

19. Prove that the volume of cone of maximum value incribed in a sphere of given radius, is 8/27 tiems the volume of the sphere.

20. Verify Rolle's theorem for the function $f(x) = 2x^3 + x^2 - 4x - 2$ when $-½ \leq x \leq \sqrt{2}$.

21. verify mean value theorem for the function $f(x) =(x-1)(x-2)(x-3)$, when $0 \leq x \leq$
4.

22. Prove that $\log_e(1+x) < x$, when $x > 0$.

# Exercise - 6

1. The function $f(x) = \frac{\log(1+ax) - \log(1-bx)}{x}$ is undefined at $x = 0$, the value which should be assigned to f at x=0, so that it is continuous at $x = 0$ is

(a) a-b (b) $\frac{a+b}{2}$ (c) a+b (d) $\log_e(a+b)$

2. The value of k which makes $(x) = x \sin\frac{1}{x}, x \neq 0$

$= k, x = 0$

continuous at $x = 0$ is

(a) 8 (b) 1 (c) -1 (d) None of these

3. The value of f(0), so that the function $f(x) = \frac{\sqrt{1+x} - \sqrt{1+x}}{x}$ is continuous at x=0, is

(a) 2 (b) 1/4 (c) 1/6 (d) 1/3

4. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ then $\frac{dy}{dx} =$

(a) $\frac{y}{y+1}$ (b) $\frac{y}{y-1}$ (c) $\frac{y}{1-y}$ (d) None of these

5. If $y = \log\{\log(\log x)\}$ then $\frac{dy}{dx} =$

(a) $\frac{1}{\log(\log x)}$ (b) $\frac{1}{x\log x \cdot \log(\log x)}$ (c) $\frac{1}{x\log(\log x)}$ (d) None of these

6. If $X^x = e^{x-y}$ then $\frac{dy}{dx} =$

(a) $\frac{\log x}{(1+\log x)^2}$ (b) Non defined (c) $\frac{1+x}{1+\log x}$ (d) $\frac{1-\log x}{1+\log x}$

7. If $y = e^{3\log x}m$ then $\frac{dy}{dx}$ is equal to

(a) $3x^2$ (b) 2logx (c) $\frac{3y}{x}$ (d) 3xy

8. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r.t. $\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is

(a) -1 (b) 1 (c) 2 (d) ½

9. $\frac{d}{dx}\left[\log\sqrt{\frac{1-\cos x}{1+\cos x}}\right]$ equals -

(a) Secx (b) Cosec x (c) Cosec $^x/_2$ (d) Sec $^x/_2$

10. Let f be differentiable for all x. If f(1) = -2 and f'(x) > 2 for $x \in [1,6]$, then

(a) f(6) > 8 (b) f(6) < 8 (c) f(6) < 5 (d) f(6) = 5

11. The rate of change of area of a circle with respect to its radius r at r = 6cm. is

(a) $12\pi$ (b) $11\pi$ (c) $10\ \pi$ (d) $8\pi$

12. The interval on which the function $f(x) = 2x^2 + 9x^2 + 12x - 1$ is decreasing is :

(a) $[-1, \propto]$ (b) [-2, -1] (c) $[-\propto, -2]$ (d) [-1, 1]

13. The angle between curves $y^2 = x$ and $x^2 = x$ and $x^2 = y$ at (1,1) is :

(a) $\tan^{-1} 4/3$ (b) $\tan^{-1} 3/4$ (c) $90^0$ (d) $45^0$

14. If x+y = k is normal to $y^2 = 12x$ then k is -

(a) 3 (b) 9 (c) -9 (d) -3

15. If the normal to the curve y = f(x) at the point (3, 4) makes an angle $3\pi/4$ with the positive x-axis the f'(3) =

(a) -1 (b) -3/4 (c) 4/3 (d) 1

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